JMPSC Division 2 Round 2

Junior Mathematicians' Problem Solving Competition

August 6th, 2022

---

1. This is a eighteen question free-response test. Each question has exactly one integer answer.
2. You have 60 minutes to complete the test.
3. You will receive 7 points for each correct answer, and 0 points for each problem left unanswered or incorrect.
4. Figures are not necessarily drawn to scale.
5. No aids are permitted other than scratch paper, graph paper, rulers, and erasers, compasses, and pencils. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. When you finish the exam, please stay in the Google meets for further instructions.

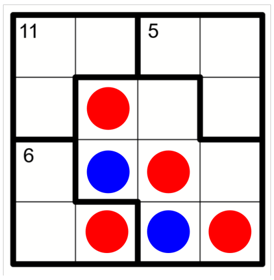
1 Division 2 R2

Problem 1

John arranges his coins into stacks. He has 4 times as many coins as stacks. If he combines the first four stacks into one, he has 5 times as many coins as stacks. How many coins does John have?

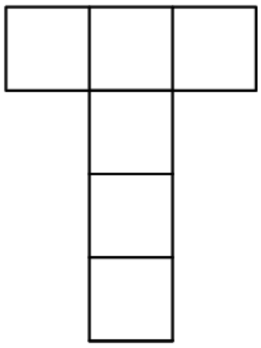
Problem 2

Fill in each cell with an integer from 1 to 4 such that no digit repeats in any row or column. Additionally, the sum of the numbers in each of the three L-shaped cages is given. If the sum of the cells with red dots is a and the sum of the cells with blue dots is b, compute $3a + 7b$.



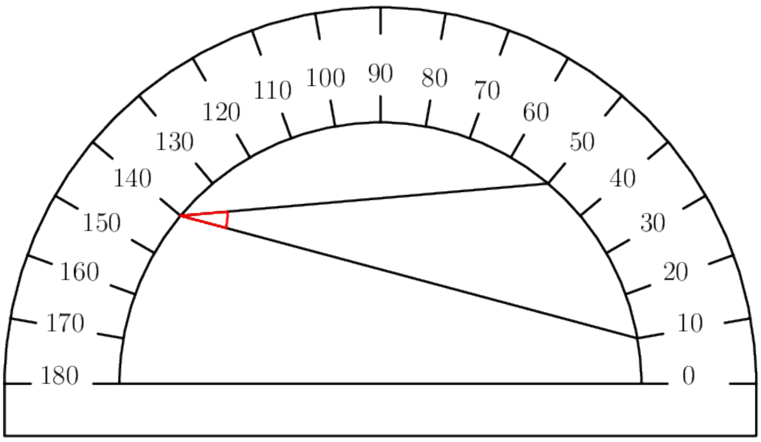
Problem 3

The numbers 1 to 6 are placed in the T-shaped grid below so that any two numbers in adjacent cells form a two-digit prime number in some order. Find the **product** of the numbers in the top row.



Problem 4

David doesn't know how to use the following semicircular protractor. Help him find the measure of the indicated red angle, in degrees.



Problem 5

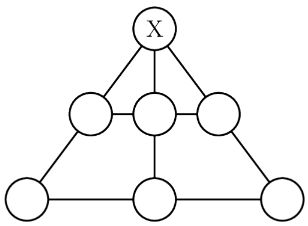
Jordan makes several cuts to a 2" × 2" × 8" stick of butter, parallel to the 2" × 2" faces, until the total surface area of the resulting slices of butter is twice as it was before. How many cuts did Jordan make?

Problem 6

A cone is placed on a table, with its base flat on the surface. When it is looked at from the side, it looks like an equilateral triangle with side length 6. The volume of the cone may be expressed as $A\pi$ for some constant A. What is A^2?

Problem 7

Distinct digits from 1 to 7 inclusive, are filled in this magic triangle such that the sum of 3 numbers in a line, is constant.



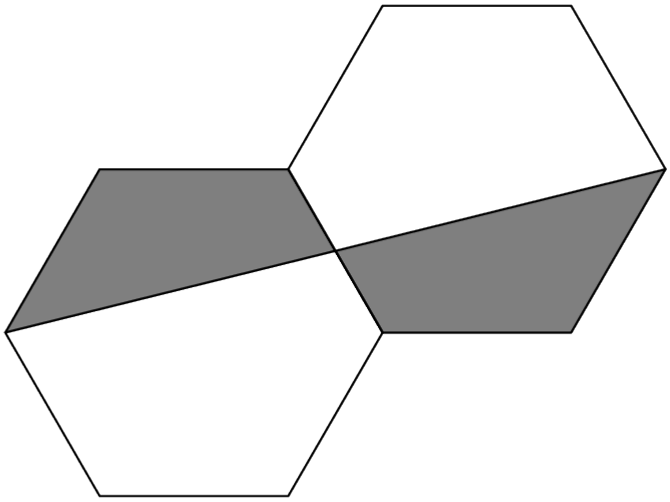
What is the sum of all possible values of X?

Problem 8

There exist three digits a, b, and c for which the two-digit positive integers $\overline{ab}$, $\overline{bc}$, and $\overline{ca}$ sum to a perfect square. What is $a+b+c$?

Problem 9

In the diagram below, both regular hexagons have area 18. What is the total area of the shaded region?

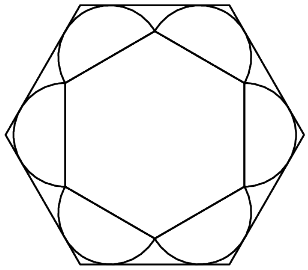


Problem 10

Let $ABCD$ be a square of side length 12. There exists points X and Y inside this square such that $AXCY$ is a rhombus, and $ABCX$ is a concave quadrilateral with area 12. What is XY^2?

Problem 11

A small hexagon with side length 2 is drawn, and semicircles with radius 1 are placed on its exterior, with their bases on the sides of the hexagon. A larger hexagon fits around the figure as shown:



If the side length of the larger hexagon can be expressed as $\frac{a}{\sqrt{b}}$, where b is square-free, what is $a+b$?

Problem 12

Let $ABCDEFG$ be a regular heptagon, and let H be the reflection of A over B. If $\angle FCH = \left(\frac{a}{b}\right)^\circ$ where a and b are relatively prime positive integers, what is $a+b$?

Problem 13

Negative real numbers a and b satisfy

$$\frac{1}{a}+\frac{1}{b}=\frac{a-b}{a+b}$$

$$\frac{1}{a-b}+\frac{1}{a+b}=\frac{1}{ab}.$$

Find a^2.

Problem 14

Determine the smallest positive integer n such that among the positive integers between n and $(n+10)$, inclusive,

- there are five multiples of 2.
- there are three multiples of 3.
- there are three multiples of 5.
- there is one multiple of 7.

Problem 15

Consider equilateral triangle ABC with side length 6. Let circle ω have radius 1 and center O, and suppose O lies on segment $\overline{AC}$ such that $OA = 2$. Let the tangent from B to ω closer to $\overline{BC}$ intersect the tangents from A to ω at X and Y. Find XY^2.

Problem 16

Let $f(x)$ be a quadratic with integer coefficients. Suppose there exist positive primes $p < q$ such that $f(p) = f(q) = 87$ and $f(p+q) = 178$. Find $p^2 + q^2$.

Problem 17

How many permutations of the first 9 positive integers satisfy both of these properties?

- The leftmost even number is 6.
- The rightmost odd number is 9.

Problem 18

Isosceles trapezoid $ABCD$ with $\overline{AB} \parallel \overline{CD}$ has an incircle ω_1 tangent to BC at E. Another circle ω_2 is tangent to ω_1, CD, and also BC at F. If $BE = CF = 1$, and the area of the trapezoid is A, what is A^2?

[1] [2]