

# JMPSC Division 2 Round 1

Junior Mathematicians' Problem Solving Competition

August 6th, 2022

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1. This is a twenty question free-response test. Each question has exactly one integer answer.
2. You have 45 minutes to complete the test.
3. You will receive 3 points for each correct answer, and 0 points for each problem left unanswered or incorrect.
4. Figures are not necessarily drawn to scale.
5. No aids are permitted other than scratch paper, graph paper, rulers, and erasers, compasses, and pencils. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. When you finish the exam, please stay in the Google meets for further instructions.

## 1 Division 2 R1

### Problem 1

What number should replace the ■ to make the equation below true?

$$20 + \blacksquare + 22 + \blacksquare = 20 \times 22$$

### Problem 2

Barbara has 35 biscuits, while Larry has 10. How many biscuits should Barbara give Larry so that Barbara has twice as many biscuits as Larry?

### Problem 3

If a regular pentagon and a regular hexagon of equal side lengths were perfectly joined along an edge, how many edges would the resulting polygon have? Assume that the two shapes do not overlap.

### Problem 4

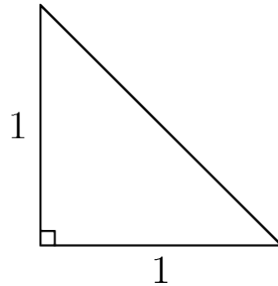
Five positive integers sum to 6. What is their product?

### Problem 5

Increasing  $n$  by 75% gives the same result as adding 21 to  $n$ . What is this common result?

**Problem 6**

What is the maximum number of triangular tiles (shown below) that can fit (without overlap) in a  $7 \times 7$  square?

**Problem 7**

What is the value of  $2\sqrt{2\sqrt{2}}$  raised to the power of 4?

**Problem 8**

Suppose  $a, b$  and  $c$  are numbers such that  $a$  is 3 times the value of  $b$ , and  $a$  is 6 times the value of  $c$ . What is the value of  $\frac{ab}{c^2}$ ?

**Problem 9**

Suppose there are  $x$  seconds in 40 hours and  $y$  hours in 40 seconds. What is the value of the product  $xy$ ?

**Problem 10**

Compute

$$\frac{10^4 - 1}{9} + \frac{10^4 - 1}{11}.$$

**Problem 11**

In an increasing sequence of 15 consecutive positive integers, the first term is half the size of the last term. Find the sum of the first and last terms.

**Problem 12**

A long piece of string is evenly divided into 6 portions. Esmeralda remarks that had the string been evenly divided into 5 portions, each portion would be 7 centimeters longer. How long was the original piece of string, in centimeters?.

**Problem 13**

What is the tens digit of the sum below?

$$1 + 12 + 123 + \cdots + 123456789$$

**Problem 14**

A large number  $q$  is divisible by the first 10 positive integers. How many of the next 10 positive integers must  $q$  be divisible by?

**Problem 15**

Patricia's Peanuts sells Miniature Boxes and Jumbo Boxes of peanuts. Beatrice buys multiple Miniature Boxes and gets 147 peanuts, while Bethany buys the same number of Jumbo Boxes and gets 203 peanuts. How many peanuts are in a Jumbo Box?

**Problem 16**

In a certain city, there are 2460 people who each own one instrument, and each of these people have their instrument tuned once every 6 months. Every tuner in this city tunes 5 instruments per month. How many tuners are there in this city?

**Problem 17**

A sign for a sale reads "AB days left!" where  $A$  and  $B$  are digits forming a two-digit number. A prankster rearranges and adds to the letters and digits on the sign to read "A days and B weeks left!", and surprisingly, the sign still reads the same number of days. What is the largest possible number of days left on the sale?

**Problem 18**

A rectangle with perimeter 60 is divided into two smaller rectangles with perimeters 32 and 48. What was the area of the original rectangle?

**Problem 19**

Line segment  $\overline{AB}$  of length 20 is rotated  $270^\circ$  about a pivot point  $O$  on  $\overline{AB}$  between  $A$  and  $B$  with  $AO = 4$  and  $OB = 16$ . If the total area of the region is  $A\pi$ , what is  $A$ ?

**Problem 20**

Suppose  $x$  and  $y$  are real numbers satisfying

$$\begin{cases} x^3 - y^3 = 493. \\ x^2y - y^2x = 50. \end{cases}$$

What is the positive difference between  $x$  and  $y$ ?

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<sup>1</sup>The publication, reproduction or communication of the problems or solutions of all JMPSC exams during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

<sup>2</sup>The team on the Junior Mathematician's Problem Solving Competition (JMPSC) reserves the right to re-examine students before deciding whether to grant official status to their scores.