JMPSC Division 1 Round 2

Junior Mathematicians' Problem Solving Competition

August 6th, 2022

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1. This is a eighteen question free-response test. Each question has exactly one integer answer.
2. You have 60 minutes to complete the test.
3. You will receive 7 points for each correct answer, and 0 points for each problem left unanswered or incorrect.
4. Figures are not necessarily drawn to scale.
5. No aids are permitted other than scratch paper, graph paper, rulers, and erasers, compasses, and pencils. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. When you finish the exam, please stay in the Google meets for further instructions.

1 Division 1 R2

Problem 1

John arranges his coins into stacks. He has 4 times as many coins as stacks. If he combines the first four stacks into one, he has 5 times as many coins as stacks. How many coins does John have?

Problem 2

There exist relatively prime positive integers m and n for which:

$$\sqrt{51+\frac{1}{49}}=\frac{m}{n}.$$

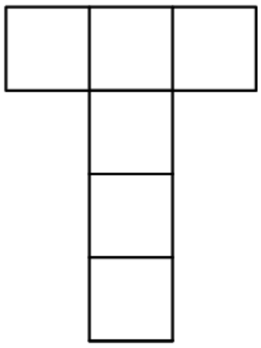
What is $m+n$?

Problem 3

Leonardo has a bookshelf with 9 rows that contain 146 books in total. Leonardo removes some books from each row so that fewer than half of the books remain on each row. What is the maximum possible number of books left on the bookshelf?

Problem 4

The numbers 1 to 6 are placed in the T-shaped grid below so that any two numbers in adjacent cells form a two-digit prime number in some order. Find the **product** of the numbers in the top row.



Problem 5

Let a, b, and c be distinct positive integers such that $\sqrt{a} + \sqrt{b} = \sqrt{c}$ and c is not a perfect square. What is the least possible value of $a + b + c$?

Problem 6

Two perpendicular chords of a circle ω have lengths 8 and 10 and intersect at a point P. The distance from P to the center of the circle is $\sqrt{19}$. If the area of the circle is $A\pi$, what is A?

Problem 7

Let $ABCDEFG$ be a regular heptagon, and let H be the reflection of A over B. If $\angle FCH = \left(\frac{a}{b}\right)^\circ$ where a and b are relatively prime positive integers, what is $a + b$?

Problem 8

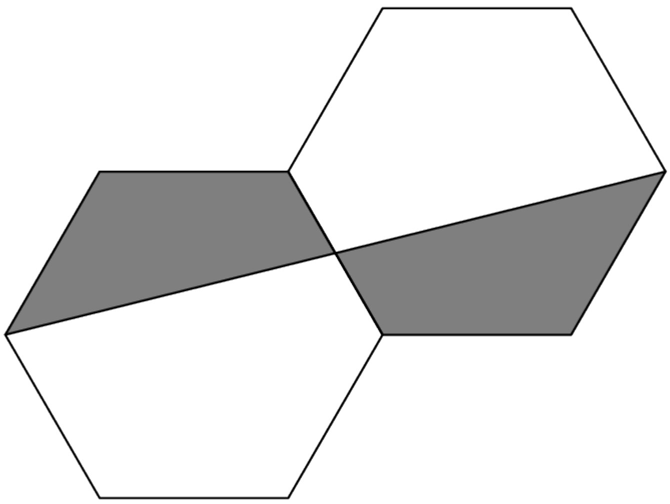
Call a positive integer N *quirky* if the remainder when the number of divisors of N is divided by 4 is 2. How many divisors of 210^9 are *quirky*?

Problem 9

Let $f(x)$ be a quadratic with integer coefficients. Suppose there exist positive primes $p < q$ such that $f(p) = f(q) = 87$ and $f(p+q) = 178$. Find $p^2 + q^2$.

Problem 10

In the diagram below, both regular hexagons have area 18. What is the total area of the shaded region?



Problem 11

What is the smallest positive integer n such that

- the sum of digits of n equals 30, and
- the sum of digits of $2n$ equals 15?

Problem 12

8 children are sitting in a line. One by one, a child is picked at random to exit the line. They walk left or right to minimize the number of other children they pass during their exit, and pat all the heads of the other children they walk past. (If a child is on the edge of the line, they leave without patting any heads.) The probability exactly 9 heads have been pat after 4 children exit the line may be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a+b$?

Problem 13

Given that a is an integer, and:

$$\frac{\frac{196^2}{a}-97}{a}$$

is a perfect square, find the value of that perfect square.

Problem 14

Find the number of ordered pairs of integer (m,n) such that

$$n^2+m^3=mn$$

and $|m|,|n| \leq 100$.

Problem 15

Isosceles trapezoid $ABCD$ with $\overline{AB} \parallel \overline{CD}$ has an incircle ω_1 tangent to BC at E. Another circle ω_2 is tangent to ω_1, CD, and also BC at F. If $BE = CF = 1$, and the area of the trapezoid is A, what is A^2?

Problem 16

The positive integers between 1 and 8, inclusive, are placed on the vertices of a cube. An operation consists of swapping **both** pairs of numbers diagonally across each other on one face. Starting from a fixed position, how many different labels of the cube's vertices are achievable after performing some number of operations on the cube?

Problem 17

Determine the integer closest in value to:

$$\frac{1}{10} \times \left(\sqrt{6^2 - 3.00^2} + \sqrt{6^2 - 3.01^2} + \sqrt{6^2 - 3.02^2} + \cdots + \sqrt{6^2 - 5.99^2}\right).$$

Problem 18

Let $ABCDE$ be a convex pentagon such that $ABCE$ is a rectangle, $AB = 150$, and $AE = DE = 180$. Let F be the point on the angle bisector of $\angle AED$ such that $\overline{CE} \perp \overline{DF}$. Let $\overleftrightarrow{BF}$ intersect $\overline{CE}$ at K and $\overline{DE}$ at L. If $CK = 70$, find EL.

[1] [2]

[1] The publication, reproduction or communication of the problems or solutions of all JMPSC exams during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

[2] The team on the Junior Mathematician's Problem Solving Competition (JMPSC) reserves the right to re-examine students before deciding whether to grant official status to their scores.