JMPSC Division 1 Round 1

Junior Mathematicians' Problem Solving Competition

August 6th, 2022

---

1. This is a twenty question free-response test. Each question has exactly one integer answer.
2. You have 45 minutes to complete the test.
3. You will receive 3 points for each correct answer, and 0 points for each problem left unanswered or incorrect.
4. Figures are not necessarily drawn to scale.
5. No aids are permitted other than scratch paper, graph paper, rulers, and erasers, compasses, and pencils. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. When you finish the exam, please stay in the Google meets for further instructions.

1 Division 1 R1

Problem 1

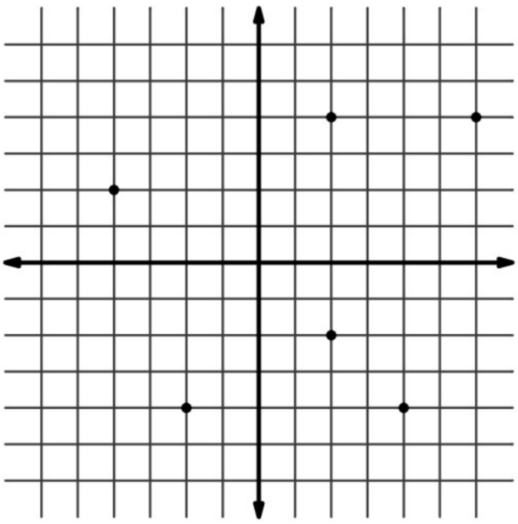
When two one-digit numbers are multiplied together, the result is greater than 30 and less than 35. What is the sum of these two numbers?

Problem 2

Six points are plotted on the coordinate grid below. Five of the points are

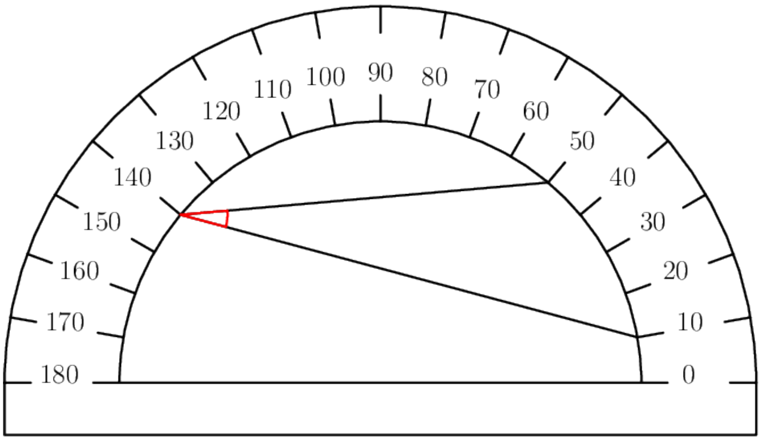
$$(9,6),\ (3,6),\ (6,-6),\ (-6,3),\ \text{and}\ (3,-3).$$

If the sixth point is at (a,b), what is the value of ab?



Problem 3

David doesn't know how to use the following semicircular protractor. Help him find the measure of the indicated red angle, in degrees.



Problem 4

Suppose there are x seconds in 40 hours and y hours in 40 seconds. What is the value of the product xy?

Problem 5

Using a 12-inch ruler, Ankit measures the side lengths of a pentagon and finds its perimeter to be 26 inches. He then realizes that he used the wrong end of the ruler, so each side length should be 12 inches minus the length he got when measuring it. What is the actual perimeter of the pentagon, in inches?

Problem 6

Jordan makes several cuts to a $2'' \times 2'' \times 8''$ stick of butter, parallel to the $2'' \times 2''$ faces, until the total surface area of the resulting slices of butter is twice as it was before. How many cuts did Jordan make?

Problem 7

What is the greatest integer value of n such that there are exactly 3 perfect squares between 7 and n, inclusive?

Problem 8

A large number q is divisible by the first 10 positive integers. How many of the next 10 positive integers must q be divisible by?

Problem 9

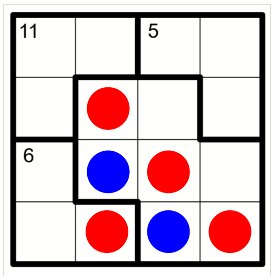
Compute $\sqrt{96 \times 98 - 71 \times 73}$.

Problem 10

In a certain city, there are 2460 people who each own one instrument, and each of these people have their instrument tuned once every 6 months. Every tuner in this city tunes 5 instruments per month. How many tuners are there in this city?

Problem 11

Fill in each cell with an integer from 1 to 4 such that no digit repeats in any row or column. Additionally, the sum of the numbers in each of the three L-shaped cages is given. If the sum of the cells with red dots is a and the sum of the cells with blue dots is b, compute $3a + 7b$.



Problem 12

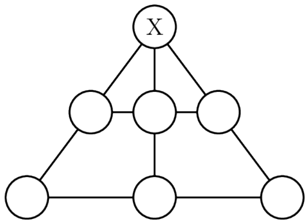
There exist three digits a, b, and c for which the two-digit positive integers $\overline{ab}$, $\overline{bc}$, and $\overline{ca}$ sum to a perfect square. What is $a + b + c$?

Problem 13

A rectangle with perimeter 60 is divided into two smaller rectangles with perimeters 32 and 48. What was the area of the original rectangle?

Problem 14

Distinct digits from 1 to 7 inclusive, are filled in this magic triangle such that the sum of 3 numbers in a line, is constant.



What is the sum of all possible values of X?

Problem 15

Line segment $\overline{AB}$ of length 20 is rotated 270° about a pivot point O on $\overline{AB}$ between A and B with $AO = 4$ and $OB = 16$. If the total area of the region is $A\pi$, what is A?

Problem 16

Suppose x and y are real numbers satisfying

$$\begin{cases} x^3 - y^3 = 493. \\ x^2y - y^2x = 50. \end{cases}$$

What is the positive difference between x and y?

Problem 17

Let $\mathcal{R}$ be a rectangle. The bisector of one of the right angles of $\mathcal{R}$ divides $\mathcal{R}$ into two shapes: a triangle with area 6 and a quadrilateral with area 12. If the perimeter of the rectangle is P, what is P^2?

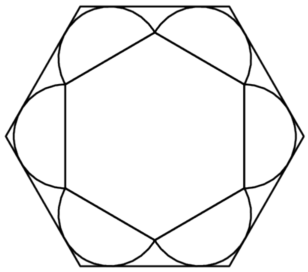
Problem 18

Determine the smallest positive integer n such that among the positive integers between n and $(n+10)$, inclusive,

- there are five multiples of 2.
- there are three multiples of 3.
- there are three multiples of 5.
- there is one multiple of 7.

Problem 19

A small hexagon with side length 2 is drawn, and semicircles with radius 1 are placed on its exterior, with their bases on the sides of the hexagon. A larger hexagon fits around the figure as shown:



If the side length of the larger hexagon can be expressed as $\frac{a}{\sqrt{b}}$, where b is square-free, what is $a+b$?

Problem 20

How many permutations of the first 9 positive integers satisfy both of these properties?

- The leftmost even number is 6.
- The rightmost odd number is 9.

[1] [2]

[1]The publication, reproduction or communication of the problems or solutions of all JMPSC exams during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

[2]The team on the Junior Mathematician's Problem Solving Competition (JMPSC) reserves the right to re-examine students before deciding whether to grant official status to their scores.