JMPSC Solutions Document

Junior Mathematicians' Problem Solving Competition

July 10th, 2021

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1. This is a dev version of the test.

1 Division 1 R1

Problem 1

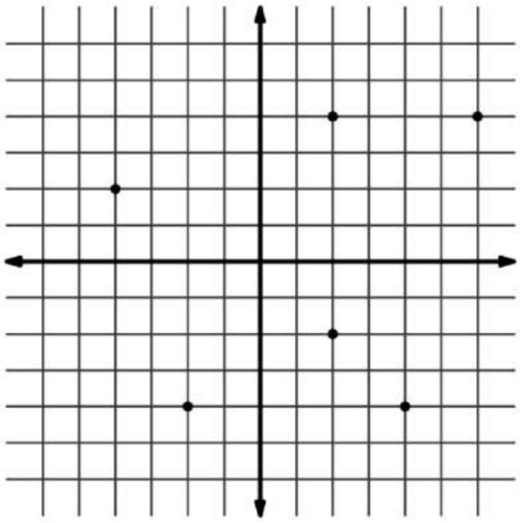
Author: bissue
Problem: When two one-digit numbers are multiplied together, the result is greater than 30 and less than 35. What is the sum of these two numbers?
Answer: 12
Solution: Notice that of the numbers 31 to 34, only 32 can be written as the product of two digits, $8 \cdot 4$. The sum of these is 12.

Problem 2

Author: Jacob Khohayting
Problem: Six points are plotted on the coordinate grid below. Five of the points are

$$(9,6),\ (3,6),\ (6,-6),\ (-6,3),\ \text{and}\ (3,-3).$$

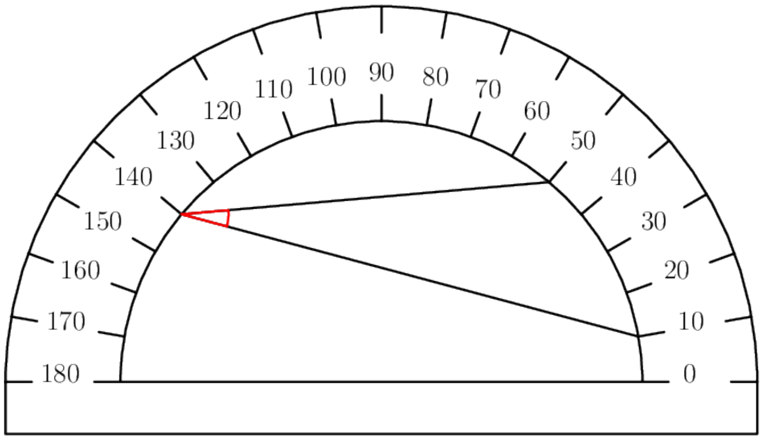
If the sixth point is at (a,b), what is the value of ab?



Answer: 18
Solution: Of the points listed, no point has both negative x and y-coordinates. However, there is a point in the third quadrant. Since we can figure out each square is $\frac{3}{2}$ units by using the other points, the lower left point must be $(-3,-6)$ for a product of 18.

Problem 3

Author: David Altizio
Problem: David doesn't know how to use the following semicircular protractor. Help him find the measure of the indicated red angle, in degrees.



Answer: 20
Solution: By the inscribed angle theorem, an inscribed angle's measure is half the measure of the arc it subtends. Since the arc it subtends has measure 40 degrees, the red angle has measure 20 degrees.

Problem 4

Author: Tiger Zhang
Problem: Suppose there are x seconds in 40 hours and y hours in 40 seconds. What is the value of the product xy?
Answer: 1600
Solution: We can write the variables in unit notation:

$$x = 40\,\frac{\text{hours}}{\text{second}}$$

and

$$y = 40\,\frac{\text{seconds}}{\text{hour}}.$$

Multiplying the two, the units cancel out and we are left with $40 \cdot 40 = 1600$.

Problem 5

Author: Isaac Li

Problem: Using a 12-inch ruler, Ankit measures the side lengths of a pentagon and finds its perimeter to be 26 inches. He then realizes that he used the wrong end of the ruler, so each side length should be 12 inches minus the length he got when measuring it. What is the actual perimeter of the pentagon, in inches?

Answer: 34

Solution: Suppose Ankit measures each side is x_i where $1 \le i \le 5$ then it is actually expressed as $12 - x_i$. So,

$$x_1 + x_2 + x_3 + x_4 + x_5 = 26.$$

So, what we want is

$$(12 - x_1) + (12 - x_2) + (12 - x_3) + (12 - x_4) + (12 - x_5).$$

This is clearly equivalent to

$$60 - (x_1 + x_2 + x_3 + x_4 + x_5)$$
$$= 60 - 26 = 34.$$

Problem 6

Author: Ari Wang
Problem: Jordan makes several cuts to a 2" × 2" × 8" stick of butter, parallel to the 2" × 2" faces, until the total surface area of the resulting slices of butter is twice as it was before. How many cuts did Jordan make?
Answer: 9
Solution: We calculate the original surface area to be 72. Notice that every cut makes two new faces of area 4, so it increases the total area by 8. To double the area, we must increase it by 72, which is $8 \cdot 9$. Thus, Jordan cuts the butter 9 times.

Problem 7

Author: bissue
Problem: What is the greatest integer value of n such that there are exactly 3 perfect squares between 7 and n, inclusive?
Answer: 35
Solution: If n is chosen to be 36 or greater, then there would be at least 4 perfect squares between 7 and n, those being:

$$\{9,\ 16,\ 25,\ 36\}$$

However, if n were chosen to be exactly 35, the only perfect squares between 7 and n would be $\{9,\ 16,\ 25\}$, which is exactly 3, as desired. Thus, the greatest possible value of n is 35.

Problem 8

Author: Ari Wang
Problem: A large number q is divisible by the first 10 positive integers. How many of the next 10 positive integers must q be divisible by?
Answer: 5
Solution: Notice that 4 of the next 10 integers are primes, which are clearly unnecessary. For the rest:

- 12: q is divisible by 3 and 4, which are relatively prime, so it must be divisible by 12.
- 14: 7 and 2 work.
- 15: 3 and 5 work.
- 16: The highest power of two that q must contain is 8. Because 16 is a higher power of two than 8, q is not necessarily divisible by it.
- 18: 9 and 2 work.
- 20: 4 and 5 work.

Thus, 5 of the integers are necessary.

Problem 9

Author: Souradip Das, David Altizio
Problem: Compute $\sqrt{96 \times 98 - 71 \times 73}$.
Answer: 65
Solution: Notice that $96 \times 98 = 97 \times 97 - 1$, and $71 \times 73 = 72 \times 72 - 1$, so the expression is equal to $\sqrt{97^2 - 72^2} = \sqrt{(97+72)(97-72)} = \sqrt{169 \times 25} = 5 \times 13 = 65$.

Problem 10

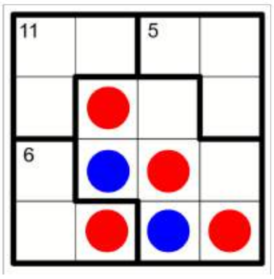
Author: bissue
Problem: In a certain city, there are 2460 people who each own one instrument, and each of these people have their instrument tuned once every 6 months. Every tuner in this city tunes 5 instruments per month. How many tuners are there in this city?
Answer: 82
Solution: Suppose there are t tuners in the city. Then since each tuner tunes 5 instruments a month, all t tuners tune $5t$ instruments a month, and thus $30t$ instruments every 6 months.

On the other hand, each of 2460 people have their instruments tuned once every 6 months, so 2460 instruments are tuned every 6 months. Therefore,

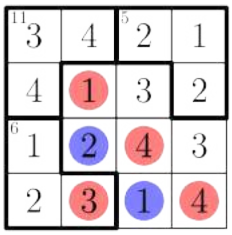
$$30t = 2460 \implies t = 82.$$

Problem 11

Author: David Altizio
Problem: Fill in each cell with an integer from 1 to 4 such that no digit repeats in any row or column. Additionally, the sum of the numbers in each of the three L-shaped cages is given. If the sum of the cells with red dots is a and the sum of the cells with blue dots is b, compute $3a+7b$.



Answer: 57
Solution: Notice 11 can only be filled in with 4, 4, and 3. So, the two boxes on the top row can be filled in with 2 and 1. So, the other box with the L–shape with a sum of 5. Then, the other box must be 2. Similarly, we can find the the left boxes are 2 and 1 and the other box in the L–shape with a sum of 6 is 3. So, the two empty cells in the second row from the top is 3 and 1. Notice the right cell must be 3 and the left cell is 1. So, the blue cell in the second row from the bottom is 2. The the left bottom square must be a 2 and the square above it is 1. So, the empty squares in the second row from the bottom must be 4 or 3. So, the right most square in the row is 3 and the square to the left of it must be 4. Then, similarly, we can find the right most square on the bottom row is 4 and the square left to it is 1. Thus, we obtain our final square,



Thus, $a=12$ and $b=3$, so our final result is $3a+7b=57$.

Problem 12

Author: Jacob Khohayting
Problem: There exist three digits a, b, and c for which the two-digit positive integers $\overline{ab}$, $\overline{bc}$, and $\overline{ca}$ sum to a perfect square. What is $a+b+c$?
Answer: 11
Solution: We sum $(10a+b)+(10b+c)+(10c+a) = 11(a+b+c)$. Since $a+b+c$ cannot exceed 27, the perfect square sum must be $11^2 = 121$. Thus, $a+b+c = 11$.

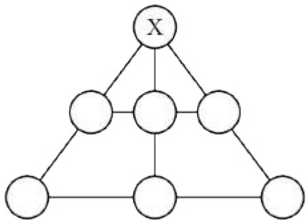
Problem 13

Author: bissue
Problem: A rectangle with perimeter 60 is divided into two smaller rectangles with perimeters 32 and 48. What was the area of the original rectangle?
Answer: 200
Solution: When the large rectangle is split, the total perimeter increases by twice the length of the cut length. Since the perimeter increases by 20, the cut length must be 10, so one of the sides of the large rectangle must be 10. This means the other side is 20, so the area of the rectangle is 200.

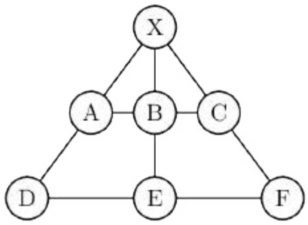
Problem 14

Author: Souradip Das
Problem:

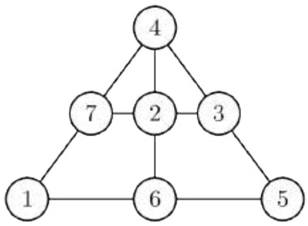
Distinct digits from 1 to 7 inclusive, are filled in this magic triangle such that the sum of 3 numbers in a line, is constant.



What is the sum of all possible values of X?
Answer: 4
Solution:

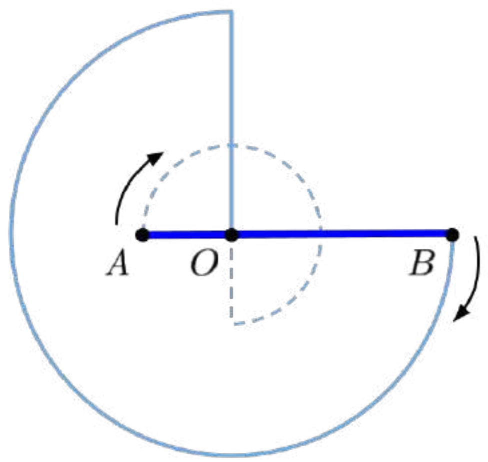


The sum of the numbers from 1 to 7 is 28. Since the sum of each line of 3 numbers is constant, it follows that $A+D=B+E=C+F$, so $28-X$ must be a multiple of 3. Additionally, $A+B+C=D+E+F$, so X must be even. Thus, the only possibility for X is 4, and we can check this works:



Problem 15

Author: Souradip Das
Problem: Line segment $\overline{AB}$ of length 20 is rotated 270° about a pivot point O on $\overline{AB}$ between A and B with $AO = 4$ and $OB = 16$. If the total area of the region is $A\pi$, what is A?
Answer: 196
Solution: The resulting configuration would be the $\frac{3}{4}$th parts of two circles, each of center O, with one rotated clockwise, and the other rotated anti-clockwise. We want to find its traced area.



However, a part of the small circle's area would be overlapped by the large one, and so the resulting area traced out by the segment, would just be $\frac{1}{4}$th of the area of the circle with radius AO, and $\frac{3}{4}$th of the area of the circle with radius OB. Hence the area would be

$$\frac{3}{4}\pi \cdot 16^2 + \frac{1}{4}\pi \cdot 4^2 = 196\pi$$

So the answer is 196.

Problem 16

Author: bissue
Problem: Suppose x and y are real numbers satisfying

$$\begin{cases} x^3 - y^3 = 493. \\ x^2y - y^2x = 50. \end{cases}$$

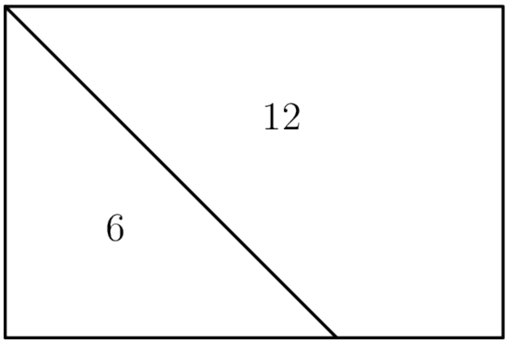
What is the positive difference between x and y?
Answer: 7
Solution: Notice that

$$\begin{aligned}(x-y)^3 &= x^3 - 3x^2y + 3y^2x - y^3 \\ &= (x^3 - y^3) - 3(x^2y - y^2x) \\ &= 493 - (3 \times 50) = 343.\end{aligned}$$

Thus, $x - y = \sqrt[3]{343} = 7$.

Problem 17

Author: David Altizio
Problem: Let $\mathcal{R}$ be a rectangle. The bisector of one of the right angles of $\mathcal{R}$ divides $\mathcal{R}$ into two shapes: a triangle with area 6 and a quadrilateral with area 12. If the perimeter of the rectangle is P, what is P^2?
Answer: 300
Solution: Let the rectangle have width w and length l. We have the following diagram:



Notice the triangle is $45-45-90$. So, $\frac{w^2}{2} = 6$ or $w^2 = 12$ and $w = 2\sqrt{3}$. Then, since the quadrilateral has area of 12 and it is a trapezoid then

$$12 = \left(\frac{l + l - 2\sqrt{3}}{2}\right) \cdot 2\sqrt{3} \implies 2l - 2\sqrt{3} = 4\sqrt{3}.$$

Hence, we obtain $l = 3\sqrt{3}$. So, the perimeter is $2l + 2w = 10\sqrt{3}$ and the answer is 300.

Problem 18

Author: bissue

Problem: Determine the smallest positive integer n such that among the positive integers between n and $(n+10)$, inclusive,

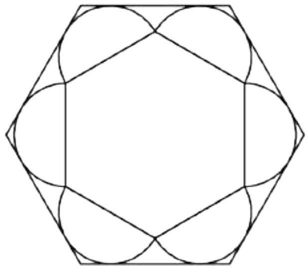
- there are five multiples of 2.
- there are three multiples of 3.
- there are three multiples of 5.
- there is one multiple of 7.

Answer: 85

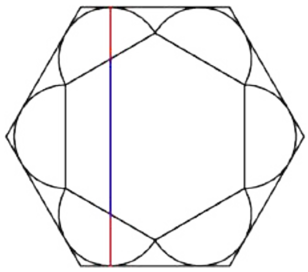
Solution: Since there are 5 multiples of 2, we know that n is odd. Also, since there are 3 multiples of 3, n must be 1 mod 3. Finally, since there are 3 multiples of 5, n must be divisible by 5. By Chinese Remainder Theorem, n is 25 mod 30. Now, we check which n work. $n = 25$ doesn't work since both 28 and 35 are in the range, $n = 55$ doesn't work since both 56 and 63 are in the range, but $n = 85$ works since only 91 is divisible by 7. Thus, $n = 85$ is the answer.

Problem 19

Author: Ethan Lee
Problem: A small hexagon with side length 2 is drawn, and semicircles with radius 1 are placed on its exterior, with their bases on the sides of the hexagon. A larger hexagon fits around the figure as shown:



If the side length of the larger hexagon can be expressed as $\frac{a}{\sqrt{b}}$, where b is square-free, what is $a+b$?
Answer: 8
Solution:



Let the side length of the large hexagon be s, so $s\sqrt{3}$ is the combined length of the colored segments. Both red segments have a length of 1, while the blue segment has a length of 3. Thus, $s\sqrt{3}=5$ so $s=\frac{5}{\sqrt{3}}$ and the answer is 8.

Problem 20

Author: bissue
Problem: How many permutations of the first 9 positive integers satisfy both of these properties?

- The leftmost even number is 6.
- The rightmost odd number is 9.

Answer: 18144
Solution: By symmetry, $\frac{1}{4}$ of all random permutations have 6 as the leftmost even number and $\frac{1}{5}$ of all random permutations have 9 as the leftmost odd number. Therefore, the answer is $9! \cdot \frac{1}{5} \cdot \frac{1}{4} = 18144$.

2 Division 1 R2

Problem 1

Author: Ethan Lee
Problem: John arranges his coins into stacks. He has 4 times as many coins as stacks. If he combines the first four stacks into one, he has 5 times as many coins as stacks. How many coins does John have?
Answer: 60
Solution: Notice that all combining four stacks does is reduce the number of stacks by 3. Then, let s be the initial number of stacks, and c be the number of coins. We have $c = 4s$ and $c = 5(s-3)$. Solving gives $s = 15$ and $c = 60$.

Problem 2

Author: bissue
Problem: There exist relatively prime positive integers m and n for which:

$$\sqrt{51 + \frac{1}{49}} = \frac{m}{n}.$$

What is $m + n$?
Answer: 57
Solution: We can simplify this expression by using Difference of Squares:

$$\sqrt{51 + \frac{1}{49}} = \sqrt{\frac{51 \times 49 + 1}{49}} = \sqrt{\frac{50^2 - 1^2 + 1}{49}} = \sqrt{\frac{50^2}{7^2}} = \frac{50}{7},$$

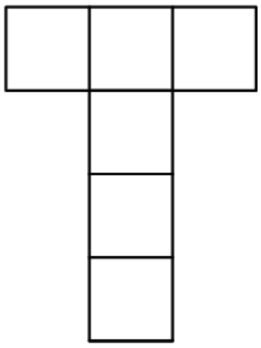
giving an answer of $50 + 7 = \boxed{57}$.

Problem 3

Author: Tiger Zhang
Problem: Leonardo has a bookshelf with 9 rows that contain 146 books in total. Leonardo removes some books from each row so that fewer than half of the books remain on each row. What is the maximum possible number of books left on the bookshelf?
Answer: 68
Solution: Let x_n be the number of books removed from the n^{th} row, and let y_n be the number of books left on the n^{th} row. Notice that if the total number of books on the n^{th} row is odd, then $x_n \geq y_n + 1$, and if $x_n + y_n$ is even, then $x_n \geq y_n + 2$. Thus, we want to maximize the number of rows with an odd number of books. It's impossible to have all 9 of the rows contain an odd number of books, because then the total number of books would be odd. However, we can get 8 rows with an odd number of books. One construction is to have 8 rows of 1 book and 1 row of 138 books, which gives the answer of 68.

Problem 4

Author: Aarush Khare
Problem: The numbers 1 to 6 are placed in the T-shaped grid below so that any two numbers in adjacent cells form a two-digit prime number in some order. Find the **product** of the numbers in the top row.



Answer: 30
Solution: The numbers 2 and 5 can only be next to 3, thus we find that the only solution occurs when the top row is $2, 3, 5$ or $5, 3, 2$, both of which yield an answer of 30.

Problem 5

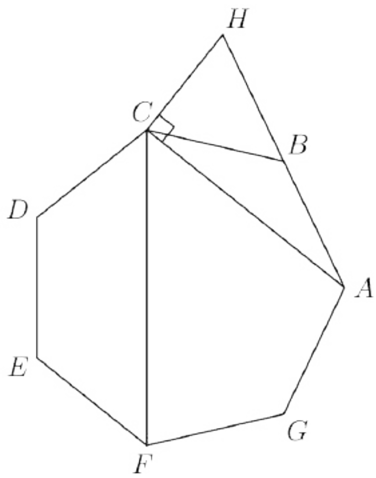
Author: Tiger Zhang
Problem: Let a, b, and c be distinct positive integers such that $\sqrt{a}+\sqrt{b}=\sqrt{c}$ and c is not a perfect square. What is the least possible value of $a+b+c$?
Answer: 28
Solution: The answer is 28, achieved by $(a,b,c)=(2,8,18)$. In that case, we have $\sqrt{2}+\sqrt{8}=\sqrt{2}+2\sqrt{2}=3\sqrt{2}=\sqrt{18}$. We will show that $\max(a,b)\geq 8$ and $\min(a,b)\geq 2$, which means $c\geq 18$, which proves that this construction is optimal.
We have $\min(a,b)\neq 1$, so $\min(a,b)\geq 2$. Square both sides to get $a+b+2\sqrt{ab}=c$. Then, ab must be a perfect square. If $\gcd(a,b)\neq 1$, then a and b are perfect square, which would make $c=(\sqrt{a}+\sqrt{b})^2$ a perfect square, contradiction. Thus, we have $\gcd(a,b)\geq 2$, and we kow that $\gcd(a,b)\mid a,b$. Notice that $\{a,b\}$ cannot be a subset of $\{\gcd(a,b), 2\gcd(a,b), 3\gcd(a,b)\}$, or ab would not be a perfect square. Thus, we have $\max(a,b)\geq 4\gcd(a,b)\geq 8$, and we are done.

Problem 6

Author: David Altizio
Problem: Two perpendicular chords of a circle ω have lengths 8 and 10 and intersect at a point P. The distance from P to the center of the circle is $\sqrt{19}$. If the area of the circle is $A\pi$, what is A?
Answer: 30
Solution: Let the chords of length 8 and 10 be $\overline{AC}$ and $\overline{BD}$ respectively, with $AP<CP$ and $BP<DP$. Now, drop an altitude to $\overline{AC}$ and $\overline{BD}$ and let their feet be K and L, respectively. If the center of the circle is O, then $PK=OL=\sqrt{r^2-25}$, so the power of P with respect to O is both r^2-19 and $41-r^2$. From this, $r^2=30$, which is our answer.

Problem 7

Author: Tiger Zhang
Problem: Let $ABCDEFG$ be a regular heptagon, and let H be the reflection of A over B. If $\angle FCH = \left(\frac{a}{b}\right)^\circ$ where a and b are relatively prime positive integers, what is $a+b$?
Answer: 997
Solution:



Notice that since $AB = HB = BC$, $\angle HCA = 90^\circ$. Now, $\angle ACF = \frac{1}{2} \cdot 2 \cdot \frac{360^\circ}{7} = \frac{360^\circ}{7}$. Thus, $\angle FCH = 90 + \frac{360}{7}$ and $\angle FCH = \frac{630^\circ + 360^\circ}{7} = \frac{990^\circ}{7}$, for an answer of 997.

Problem 8

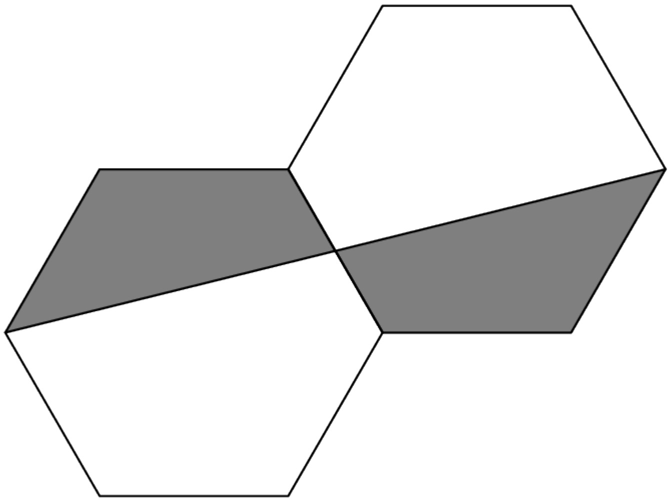
Author: Isaac Li
Problem: Call a positive integer N *quirky* if the remainder when the number of divisors of N is divided by 4 is 2. How many divisors of 210^9 are *quirky*?
Answer: 1500
Solution: Let the number of factors of N be x. Notice that if $x \equiv 2 \pmod 4$, then that is equivalent to x being divisible by 2 but not 4.
Since $210^9 = 2^9 \cdot 3^9 \cdot 5^9 \cdot 7^9$, $N = 2^a \cdot 3^b \cdot 5^c \cdot 7^d$ for some $0 \le a,b,c,d \le 9$. Then, the number of factors of N is $(a+1)(b+1)(c+1)(d+1)$.
If x is divisible by 2 but not by 4, exactly one of $a+1$, $b+1$, $c+1$, or $d+1$ must be divisible by 2 but not by 4, and the rest must be odd. The even choice may be one of 2, 6, or 10, while the odd choice may be one of 1, 3, 5, 7, 9. Thus, the answer is $4 \cdot 3 \cdot 5^3 = 1500$.

Problem 9

Author: Hanson Liu, David Altizio
Problem: Let $f(x)$ be a quadratic with integer coefficients. Suppose there exist positive primes $p < q$ such that $f(p) = f(q) = 87$ and $f(p+q) = 178$. Find $p^2 + q^2$.
Answer: 218
Solution: Define $g(x) = f(x) - 87$, so $g(p) = g(q) = 0$ and thus $g(x) = a(x-p)(x-q)$ for some integer a. Then, $g(p+q) = apq = 178 - 87 = 91$, which forces $a = 1$ and $p, q = 7, 13$ in some order. Thus, the answer is $49 + 169 = 218$.

Problem 10

Author: Ethan Lee
Problem: In the diagram below, both regular hexagons have area 18. What is the total area of the shaded region?



Answer: 12
Solution: Put both shaded regions in the same hexagon (by reflecting one region over the shared side of the hexagon). Notice that there is an empty triangle in the hexagon. Divide the hexagon into six congruent equilateral triangles. Since the empty triangle has the same base and twice the height of any equilateral triangle, it must have twice the area, or $\frac{1}{3}$ of the hexagon. Thus, the shaded region is $\frac{2}{3}$ of the hexagon, or 12.

Problem 11

Author: bissue
Problem: What is the smallest positive integer n such that

- the sum of digits of n equals 30, and
- the sum of digits of $2n$ equals 15?

Answer: 55569
Solution: If there were no regroups when adding $N+N$, $2N$ would have a digit sum of 60. However, each regroup causes a decrease of 9 in the sum's digit sum, so we know that 5 regroups occurred. This means that N has at least 5 digits, and in each position there is a regroup, so each digit in N is greater than or equal to 5. The least such N is 55569 and we can check that this works.

Problem 12

Author: Ari Wang
Problem:
8 children are sitting in a line. One by one, a child is picked at random to exit the line. They walk left or right to minimize the number of other children they pass during their exit, and pat all the heads of the other children they walk past. (If a child is on the edge of the line, they leave without patting any heads.) The probability exactly 9 heads have been pat after 4 children exit the line may be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a+b$?
Answer: 71
Solution: The key observation is that the number of heads patted by each child is independent of each other, since after a child leaves a line of n children, we will always be left with $n-1$. Now, the maximum number of head pats is $3+3+2+2=10$, so one child must have patted one less child than they could have. We have four cases:

- 2, 3, 2, 2: This is $\frac{2}{8} \cdot \frac{1}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} = \frac{1}{420}$.
- 3, 2, 2, 2: This is $\frac{2}{8} \cdot \frac{2}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} = \frac{2}{420}$.
- 2, 3, 1, 2: This is $\frac{2}{8} \cdot \frac{1}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} = \frac{1}{420}$.
- 2, 3, 2, 1: This is $\frac{2}{8} \cdot \frac{1}{7} \cdot \frac{2}{6} \cdot \frac{2}{5} = \frac{2}{420}$.

Adding the probabilities up gives $\frac{1}{70}$, for an answer of 71.

Problem 13

Author: Samuel Choi
Problem: Given that a is an integer, and:

$$\frac{\frac{196^2}{a} - 97}{a}$$

is a perfect square, find the value of that perfect square.
Answer: 144
Solution: Let our answer be b. Note that the problem rearranges itself to: Find ab if a, b are positive integers such that

$$a(ab + 97) = 2^4 \cdot 7^4.$$

Since both factors $a, ab + 97$ on the LHS are integers, $97 \nmid a$. Since $\gcd(a, ab + 97) = \gcd(a, 97) = 1$, we must have that the two facors are coprime and therefore $a = 1, 2^4, 7^4, 2^4 \cdot 7^4$. Guessing $a = 2^4$ suffices to show that $b = 144$.

Problem 14

Author: Luv Udeshi
Problem: Find the number of ordered pairs of integer (m,n) such that

$$n^2 + m^3 = mn$$

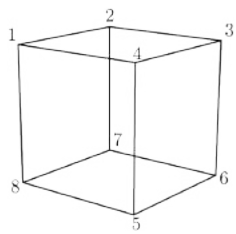
and $|m|,|n| \leq 100$.
Answer: 9
Solution: Since the equation is a quadratic in terms of n, we solve to get $n = \frac{m \pm m\sqrt{1-4m}}{2} = \frac{m}{2}\left(1 \pm \sqrt{1-4m}\right)$. Since $1 \pm \sqrt{1-4m}$ must be rational, it is an integer. We let $a = \frac{1\pm\sqrt{1-4m}}{2}$ be some integer (even or odd) divided by 2, so $n = am$.
Plugging this in, we have $a^2m^2 + m^2 = am^2$, or $m = -a^2 + a$ and $n = -a^3 + a^2$. Plugging this back into the original equation works, and in order for m and n to be integer, a must be integer as well. Thus, this is the general solution for integer a. Checking the possible range of a for $|m|,|n| \leq 100$, we find that $a = 5$ gives $(m,n) = (-20,-100)$ and $a = -4$ gives $(m,n) = (-20,80)$. However, $a = -1$ and $a = 0$ give the same solution of $(0,0)$. Thus, there are $5-(-4)+1-1 = 9$ solutions.

Problem 15

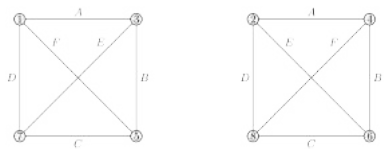
Author: Ethan Lee
Problem: Isosceles trapezoid $ABCD$ with $\overline{AB} \parallel \overline{CD}$ has an incircle ω_1 tangent to BC at E. Another circle ω_2 is tangent to ω_1, CD, and also BC at F. If $BE = CF = 1$, and the area of the trapezoid is A, what is A^2?
Answer: 192
Solution: Let the common tangent of the two circles intersect BC at P and CD at Q. By radical axis P is the midpoint of EF and therefore also the midpoint of BC. By symmetry, $PQ = PB = PC$, but also $CP = CQ$ so CPQ is equilateral. This implies that $BQ \perp QC$, so we calculate $BQ = \sqrt{BC^2 - QC^2}$. The area of the trapezoid is the sum of the bases divided by 2 times its height, which can be calculated to be $8\sqrt{3}$ by all the congruent segments, which yields 192 upon squaring.

Problem 16

Author: Ethan Lee
Problem: The positive integers between 1 and 8, inclusive, are placed on the vertices of a cube. An operation consists of swapping **both** pairs of numbers diagonally across each other on one face. Starting from a fixed position, how many different labels of the cube's vertices are achievable after performing some number of operations on the cube?
Answer: 96
Solution: Without loss of generality, label the cube as follows:



Now, assign each operation to the cube a letter which switches the pairs of numbers:



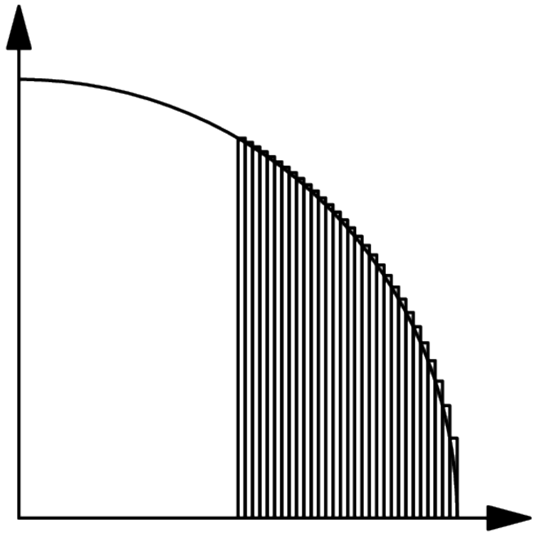
Notice that through the swaps A, B, C, and D, we can swap the odd numbers into any possible permutation. We count the states where the odd numbers are in their original positions.
If we do the operations $ABFB$, the odd numbers stay the same while in the even numbers, the 2 and 4 are swapped and the 6 and 8 are swapped. Likewise, doing operations $BCEC$ keeps the odd numbers constant while swapping 2 and 8, 4 and 6. By doing both of these operations, it is possible to swap 2 and 6, 4 and 8 from the original position. Thus, for this particular permutation for the odd numbers, we have 4 possible permutations for the even numbers. We show that these are all the possible states.
Notice that every swap we do on the original cube must keep the parity of numbers less than 5 (so 1, 2, 3, and 4, which we call *low* numbers) on each face even, as well as integers between 3 and 6 inclusive (called *middle* numbers). Through this, we can deduce that there are 4 possible states, each of which corresponds to the four we attained by doing the operation combinations $ABFB$ and $BCEC$. Since we have shown that these four are attainable and that no other permutations are attainable, there must be $4 \cdot 24 = 96$ possible permutations of the original cube.

Problem 17

Author: bissue
Problem: Determine the integer closest in value to:

$$\frac{1}{10} \times \left(\sqrt{6^2 - 3.00^2} + \sqrt{6^2 - 3.01^2} + \sqrt{6^2 - 3.02^2} + \cdots + \sqrt{6^2 - 5.99^2}\right).$$

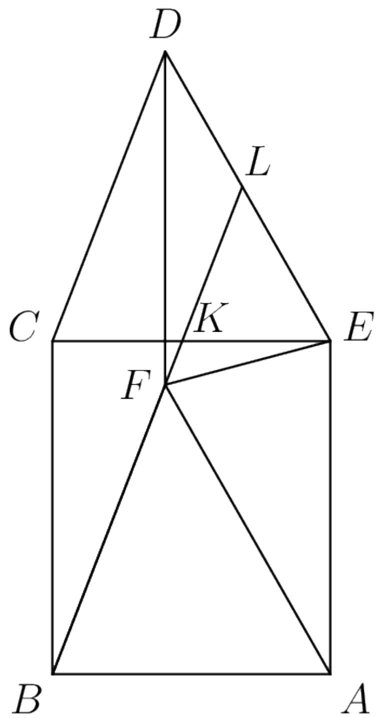
Answer: 111
Solution: Notice that the sum is actually an approximation for the area under a portion of a quarter circle:



Using some geometry, the portion of the circle is $6\pi - \frac{9\sqrt{3}}{2}$, which can be estimated to be $6 \cdot 3.14 - 4.5 \cdot 1.73$ or 11.055. From the diagram, this is a little less than the total area of the rectangles, so the rectangles have an area of about 11.1 to the nearest tenth. The answer is 111.

Problem 18

Author: Tiger Zhang
Problem: Let $ABCDE$ be a convex pentagon such that $ABCE$ is a rectangle, $AB = 150$, and $AE = DE = 180$. Let F be the point on the angle bisector of $\angle AED$ such that $\overline{CE} \perp \overline{DF}$. Let $\overleftrightarrow{BF}$ intersect $\overline{CE}$ at K and $\overline{DE}$ at L. If $CK = 70$, find EL.
Answer: 96
Solution:



Notice that $\angle FEA = \angle DFE = \angle DEF$, implying $DF = DE = EA = BC$. Thus, since $\overline{BC} \parallel \overline{DF}$, $BFDC$ is a parallelogram so $\overline{CD} \parallel \overline{BF}$. However, this then implies $\triangle KLE \sim \triangle CDE$, so $EL = DE \cdot \frac{KE}{CE} = 180 \cdot \frac{8}{15} = 96$.

3 Division 2 R1

Problem 1

Author: bissue
Problem: What number should replace the ■ to make the equation below true?

$$20 + \blacksquare + 22 + \blacksquare = 20 \times 22$$

Answer: 199
Solution: farley orz

Problem 2

Author: bissue
Problem: Barbara has 35 biscuits, while Larry has 10. How many biscuits should Barbara give Larry so that Barbara has twice as many biscuits as Larry?
Answer: 5
Solution: After Barbara gives Larry x biscuits, Barbara will have $35 - x$ and Larry will have $10 + x$. We want to solve $(35 - x) = 2(10 + x)$ so

$$35 - x = 2x = 20$$

and $x = 5$.

Problem 3

Author: bissue
Problem: If a regular pentagon and a regular hexagon of equal side lengths were perfectly joined along an edge, how many edges would the resulting polygon have? Assume that the two shapes do not overlap.
Answer: 9
Solution: Separately, the pentagon and hexagon have 11 sides in total. When they are joined, the edges with which they are joined are no longer counted. Thus, the answer is $11 - 2 = 9$.

Problem 4

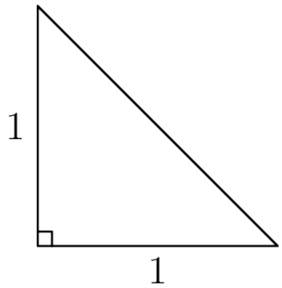
Author: David Altizio
Problem: Five positive integers sum to 6. What is their product?
Answer: 2
Solution: The five integers must be $1, 1, 1, 1, 2$, so their product is 2.

Problem 5

Author: bissue
Problem: Increasing n by 75% gives the same result as adding 21 to n. What is this common result?
Answer: 49
Solution: From the problem statement, 75% of n must be 21. Thus, $n = \frac{100}{75} \cdot 21 = 28$, so the answer is $21 + 28 = 49$.

Problem 6

Author: Ethan Lee
Problem: What is the maximum number of triangular tiles (shown below) that can fit (without overlap) in a 7×7 square?



Answer: 98
Solution: Split the 7×7 square into 49 unit squares. Each unit square can fit two tiles, so 49 of them can fit 98 tiles.

Problem 7

Author: Samuel Choi
Problem: What is the value of $2\sqrt{2\sqrt{2}}$ raised to the power of 4?
Answer: 128
Solution: Note that

$$2\sqrt{2\sqrt{2}} = 2\sqrt{2^{\frac{3}{2}}} = 2 \cdot 2^{\frac{3}{4}} = 2^{\frac{7}{4}}.$$

Taking this to the fourth power, $\left(2^{\frac{7}{4}}\right)^4 = 2^7 = 128$.

Problem 8

Author: Samuel Choi
Problem: Suppose a, b and c are numbers such that a is 3 times the value of b, and a is 6 times the value of c. What is the value of $\frac{ab}{c^2}$?
Answer: 12
Solution: Notice that $b = \frac{a}{3}$ and $c = \frac{a}{6}$ from the givens. Thus, $\frac{ab}{c^2} = \frac{a^2/3}{a^2/36} = 12$.

Problem 9

Author: Tiger Zhang
Problem: Suppose there are x seconds in 40 hours and y hours in 40 seconds. What is the value of the product xy?
Answer: 1600
Solution: We can write the variables in unit notation:

$$x = 40 \, \frac{\text{hours}}{\text{second}}$$

and

$$y = 40 \, \frac{\text{seconds}}{\text{hour}}.$$

Multiplying the two, the units cancel out and we are left with $40 \cdot 40 = 1600$.

Problem 10

Author: Tiger Zhang
Problem: Compute

$$\frac{10^4 - 1}{9} + \frac{10^4 - 1}{11}.$$

Answer: 2020
Solution: We know that $10^4 - 1 = 9999$, and dividing this by 9 and 11 respectively gives 1111 and 909. Summing these, the answer is 2020.

Problem 11

Author: Samuel Choi
Problem: In an increasing sequence of 15 consecutive positive integers, the first term is half the size of the last term. Find the sum of the first and last terms.
Answer: 42
Solution: If the first term is n, the last term is $2n$ and there are $n+1$ terms in total. Since $n+1 = 15$, $n = 14$, $2n = 28$, and the sum of the first and last terms is 42.

Problem 12

Author: bissue
Problem: A long piece of string is evenly divided into 6 portions. Esmeralda remarks that had the string been evenly divided into 5 portions, each portion would be 7 centimeters longer. How long was the original piece of string, in centimeters?.
Answer: 210
Solution: Suppose the string has length x centimeters. If the string were divided into 6 portions, each portion would have length $\frac{x}{6}$, whereas if the string were divided into 5 portions, each portion would have length $\frac{x}{5}$. Therefore,

$$\frac{x}{6} + 7 = \frac{x}{5} \implies 7 = \frac{x}{30} \implies x = 210.$$

Problem 13

Author: Ethan Lee
Problem: What is the tens digit of the sum below?

$$1 + 12 + 123 + \cdots + 123456789$$

Answer: 0
Solution: Notice that any digits above the tens place or above doesn't matter, so the answer is the same as the tens digit of $1 + 12 + 23 + \cdots + 89$. We break this sum up by ones and tens digits. Summing the ones digits, or $1 + 2 + \cdots + 9$ gives 45, and summing the tens digits, or $10 + 20 + \cdots + 80$ gives 360. The sum of these is 405, so our answer is 0.

Problem 14

Author: Ari Wang
Problem: A large number q is divisible by the first 10 positive integers. How many of the next 10 positive integers must q be divisible by?
Answer: 5
Solution: Notice that 4 of the next 10 integers are primes, which are clearly unnecessary. For the rest:

- 12: q is divisible by 3 and 4, which are relatively prime, so it must be divisible by 12.
- 14: 7 and 2 work.
- 15: 3 and 5 work.
- 16: The highest power of two that q must contain is 8. Because 16 is a higher power of two than 8, q is not necessarily divisible by it.
- 18: 9 and 2 work.
- 20: 4 and 5 work.

Thus, 5 of the integers are necessary.

Problem 15

Author: bissue
Problem: Patricia's Peanuts sells Miniature Boxes and Jumbo Boxes of peanuts. Beatrice buys multiple Miniature Boxes and gets 147 peanuts, while Bethany buys the same number of Jumbo Boxes and gets 203 peanuts. How many peanuts are in a Jumbo Box?
Answer: 29
Solution: Suppose that the number of boxes bought by each person is n. We have that $mn = 147$, $jn = 203$, where m and j are the number of peanuts in a miniature and jumbo box respectively. All of these variables are positive integers, so n must be a factor of both 147 and 203. As a consequence, n must divide the greatest common divisor of the two numbers. Evaluating, $\gcd(147, 203) = 7$, so n is either 1 or 7. Since Beatrice bought multiple, not one, miniature boxes, n must be 7 and as a result $j = 29$.

Problem 16

Author: bissue
Problem: In a certain city, there are 2460 people who each own one instrument, and each of these people have their instrument tuned once every 6 months. Every tuner in this city tunes 5 instruments per month. How many tuners are there in this city?
Answer: 82
Solution: Suppose there are t tuners in the city. Then since each tuner tunes 5 instruments a month, all t tuners tune $5t$ instruments a month, and thus $30t$ instruments every 6 months.

On the other hand, each of 2460 people have their instruments tuned once every 6 months, so 2460 instruments are tuned every 6 months. Therefore,

$$30t = 2460 \implies t = 82.$$

Problem 17

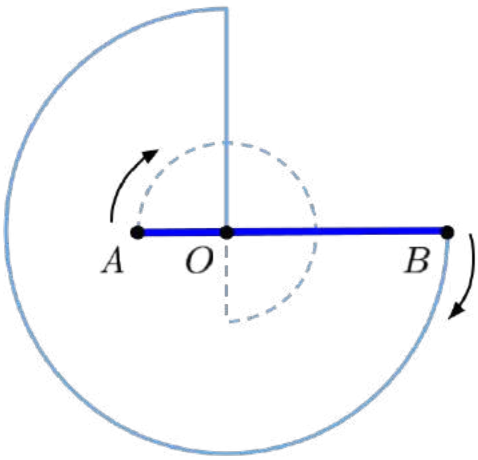
Author: Isaac Li
Problem: A sign for a sale reads "AB days left!" where A and B are digits forming a two-digit number. A prankster rearranges and adds to the letters and digits on the sign to read "A days and B weeks left!", and surprisingly, the sign still reads the same number of days. What is the largest possible number of days left on the sale?
Answer: 69
Solution: We have the equations $A + 7B = \overline{AB} = 10A + B$. Simplifying gives $6B = 9A$, and since A and B are digits, the largest possible solution is $A = 6$, $B = 9$. Then, $\overline{AB} = 69$.

Problem 18

Author: bissue
Problem: A rectangle with perimeter 60 is divided into two smaller rectangles with perimeters 32 and 48. What was the area of the original rectangle?
Answer: 200
Solution: When the large rectangle is split, the total perimeter increases by twice the length of the cut length. Since the perimeter increases by 20, the cut length must be 10, so one of the sides of the large rectangle must be 10. This means the other side is 20, so the area of the rectangle is 200.

Problem 19

Author: Souradip Das
Problem: Line segment $\overline{AB}$ of length 20 is rotated 270° about a pivot point O on $\overline{AB}$ between A and B with $AO = 4$ and $OB = 16$. If the total area of the region is $A\pi$, what is A?
Answer: 196
Solution: The resulting configuration would be the $\frac{3}{4}$th parts of two circles, each of center O, with one rotated clockwise, and the other rotated anti-clockwise. We want to find its traced area.



However, a part of the small circle's area would be overlapped by the large one, and so the resulting area traced out by the segment, would just be $\frac{1}{4}$th of the area of the circle with radius AO, and $\frac{3}{4}$th of the area of the circle with radius OB. Hence the area would be

$$\frac{3}{4}\pi \cdot 16^2 + \frac{1}{4}\pi \cdot 4^2 = 196\pi$$

So the answer is 196.

Problem 20

Author: bissue
Problem: Suppose x and y are real numbers satisfying

$$\begin{cases} x^3 - y^3 = 493. \\ x^2y - y^2x = 50. \end{cases}$$

What is the positive difference between x and y?
Answer: 7
Solution: Notice that

$$\begin{aligned}(x-y)^3 &= x^3 - 3x^2y + 3y^2x - y^3 \\ &= (x^3 - y^3) - 3(x^2y - y^2x) \\ &= 493 - (3 \times 50) = 343.\end{aligned}$$

Thus, $x - y = \sqrt[3]{343} = 7$.

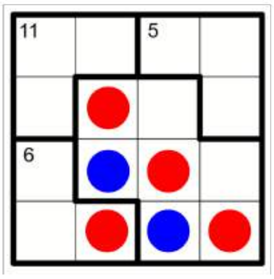
4 Division 2 R2

Problem 1

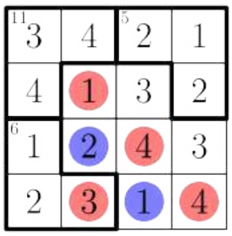
Author: Ethan Lee
Problem: John arranges his coins into stacks. He has 4 times as many coins as stacks. If he combines the first four stacks into one, he has 5 times as many coins as stacks. How many coins does John have?
Answer: 60
Solution: Notice that all combining four stacks does is reduce the number of stacks by 3. Then, let s be the initial number of stacks, and c be the number of coins. We have $c = 4s$ and $c = 5(s-3)$. Solving gives $s = 15$ and $c = 60$.

Problem 2

Author: David Altizio
Problem: Fill in each cell with an integer from 1 to 4 such that no digit repeats in any row or column. Additionally, the sum of the numbers in each of the three L-shaped cages is given. If the sum of the cells with red dots is a and the sum of the cells with blue dots is b, compute $3a+7b$.



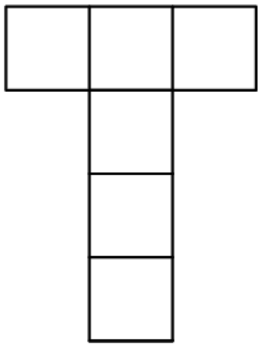
Answer: 57
Solution: Notice 11 can only be filled in with 4, 4, and 3. So, the two boxes on the top row can be filled in with 2 and 1. So, the other box with the L–shape with a sum of 5. Then, the other box must be 2. Similarly, we can find the the left boxes are 2 and 1 and the other box in the L–shape with a sum of 6 is 3. So, the two empty cells in the second row from the top is 3 and 1. Notice the right cell must be 3 and the left cell is 1. So, the blue cell in the second row from the bottom is 2. The the left bottom square must be a 2 and the square above it is 1. So, the empty squares in the second row from the bottom must be 4 or 3. So, the right most square in the row is 3 and the square to the left of it must be 4. Then, similarly, we can find the right most square on the bottom row is 4 and the square left to it is 1. Thus, we obtain our final square,



Thus, $a=12$ and $b=3$, so our final result is $3a+7b=57$.

Problem 3

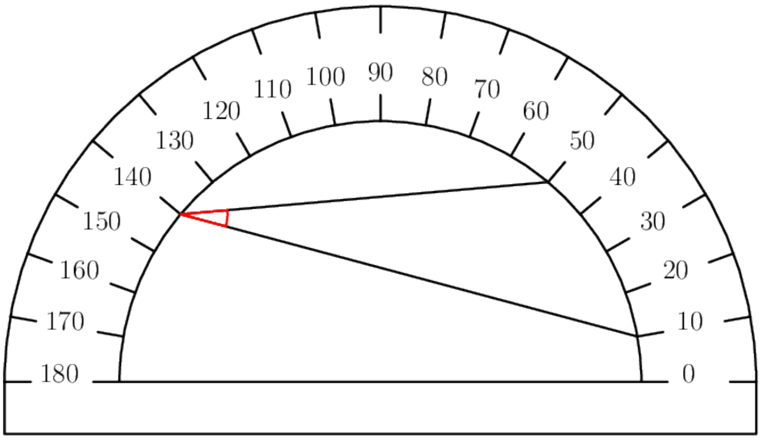
Author: Aarush Khare
Problem: The numbers 1 to 6 are placed in the T-shaped grid below so that any two numbers in adjacent cells form a two-digit prime number in some order. Find the **product** of the numbers in the top row.



Answer: 30
Solution: The numbers 2 and 5 can only be next to 3, thus we find that the only solution occurs when the top row is $2, 3, 5$ or $5, 3, 2$, both of which yield an answer of 30.

Problem 4

Author: David Altizio
Problem: David doesn't know how to use the following semicircular protractor. Help him find the measure of the indicated red angle, in degrees.



Answer: 20
Solution: By the inscribed angle theorem, an inscribed angle's measure is half the measure of the arc it subtends. Since the arc it subtends has measure 40 degrees, the red angle has measure 20 degrees.

Problem 5

Author: Ari Wang
Problem: Jordan makes several cuts to a $2'' \times 2'' \times 8''$ stick of butter, parallel to the $2'' \times 2''$ faces, until the total surface area of the resulting slices of butter is twice as it was before. How many cuts did Jordan make?
Answer: 9
Solution: We calculate the original surface area to be 72. Notice that every cut makes two new faces of area 4, so it increases the total area by 8. To double the area, we must increase it by 72, which is $8 \cdot 9$. Thus, Jordan cuts the butter 9 times.

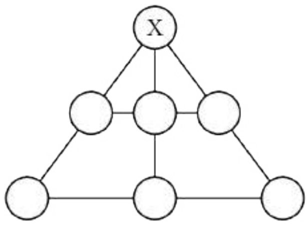
Problem 6

Author: bissue
Problem: A cone is placed on a table, with its base flat on the surface. When it is looked at from the side, it looks like an equilateral triangle with side length 6. The volume of the cone may be expressed as $A\pi$ for some constant A. What is A^2?
Answer: 243
Solution: Since the equilateral triangle must have a height of $3\sqrt{3}$, the cone must have the same height as well. Also, the base of the triangle is the diameter of the cone, or 6. So, the radius of the cone is 3, and we can compute the volume to be $9\sqrt{3}\pi$ for an answer of 243.

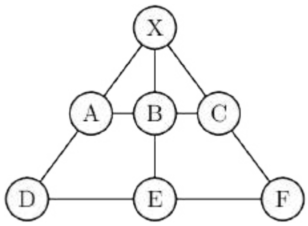
Problem 7

Author: Souradip Das
Problem:

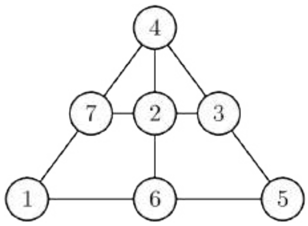
Distinct digits from 1 to 7 inclusive, are filled in this magic triangle such that the sum of 3 numbers in a line, is constant.



What is the sum of all possible values of X?
Answer: 4
Solution:



The sum of the numbers from 1 to 7 is 28. Since the sum of each line of 3 numbers is constant, it follows that $A+D=B+E=C+F$, so $28-X$ must be a multiple of 3. Additionally, $A+B+C=D+E+F$, so X must be even. Thus, the only possibility for X is 4, and we can check this works:

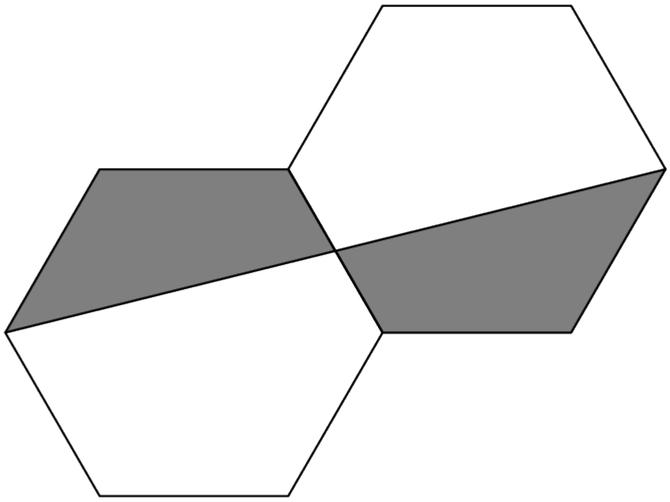


Problem 8

Author: Jacob Khohayting
Problem: There exist three digits a, b, and c for which the two-digit positive integers $\overline{ab}$, $\overline{bc}$, and $\overline{ca}$ sum to a perfect square. What is $a+b+c$?
Answer: 11
Solution: We sum $(10a+b)+(10b+c)+(10c+a) = 11(a+b+c)$. Since $a+b+c$ cannot exceed 27, the perfect square sum must be $11^2 = 121$. Thus, $a+b+c = 11$.

Problem 9

Author: Ethan Lee
Problem: In the diagram below, both regular hexagons have area 18. What is the total area of the shaded region?



Answer: 12
Solution: Put both shaded regions in the same hexagon (by reflecting one region over the shared side of the hexagon). Notice that there is an empty triangle in the hexagon. Divide the hexagon into six congruent equilateral triangles. Since the empty triangle has the same base and twice the height of any equilateral triangle, it must have twice the area, or $\frac{1}{3}$ of the hexagon. Thus, the shaded region is $\frac{2}{3}$ of the hexagon, or 12.

Problem 10

Author: bissue

Problem: Let $ABCD$ be a square of side length 12. There exists points X and Y inside this square such that $AXCY$ is a rhombus, and $ABCX$ is a concave quadrilateral with area 12. What is XY^2?

Answer: 200

Solution: Observe that since $AXCY$ is a rhombus, there is a clean symmetry between X and Y with respect to $ABCD$. In particular, the area of $ABCX$ equals the area of $ADCY$, meaning that:

$$[AXCY] = [ABCD] - [ABCX] - [ADCY] = 12^2 - 12 - 12 = 120.$$

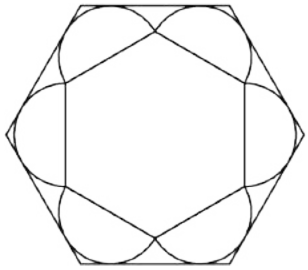
Also notice that the area of $AXCY$ is half the product of its diagonals (since $AXCY$ is a rhombus), so:

$$120 = [AXCY] = \frac{1}{2} \times AC \times XY = \frac{1}{2} \times 12\sqrt{2} \times XY \implies XY = 10\sqrt{2}.$$

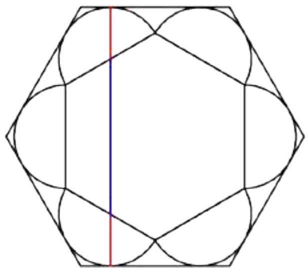
So $XY^2 = 200$.

Problem 11

Author: Ethan Lee
Problem: A small hexagon with side length 2 is drawn, and semicircles with radius 1 are placed on its exterior, with their bases on the sides of the hexagon. A larger hexagon fits around the figure as shown:



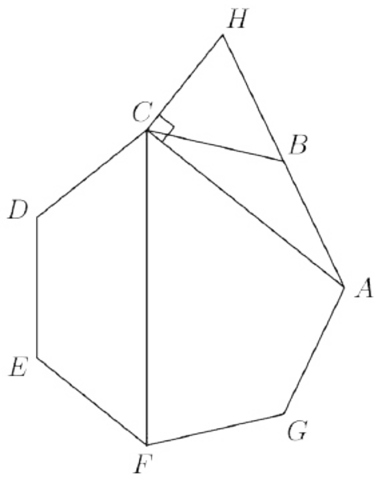
If the side length of the larger hexagon can be expressed as $\frac{a}{\sqrt{b}}$, where b is square-free, what is $a+b$?
Answer: 8
Solution:



Let the side length of the large hexagon be s, so $s\sqrt{3}$ is the combined length of the colored segments. Both red segments have a length of 1, while the blue segment has a length of 3. Thus, $s\sqrt{3}=5$ so $s=\frac{5}{\sqrt{3}}$ and the answer is 8.

Problem 12

Author: Tiger Zhang
Problem: Let $ABCDEFG$ be a regular heptagon, and let H be the reflection of A over B. If $\angle FCH = \left(\frac{a}{b}\right)^\circ$ where a and b are relatively prime positive integers, what is $a+b$?
Answer: 997
Solution:



Notice that since $AB = HB = BC$, $\angle HCA = 90^\circ$. Now, $\angle ACF = \frac{1}{2} \cdot 2 \cdot \frac{360^\circ}{7} = \frac{360^\circ}{7}$. Thus, $\angle FCH = 90 + \frac{360}{7}$ and $\angle FCH = \frac{630^\circ + 360^\circ}{7} = \frac{990^\circ}{7}$, for an answer of 997.

Problem 13

Author: Suhaan M

Problem: Negative real numbers a and b satisfy

$$\frac{1}{a}+\frac{1}{b}=\frac{a-b}{a+b}$$

$$\frac{1}{a-b}+\frac{1}{a+b}=\frac{1}{ab}.$$

Find a^2.

Answer: 4

Solution:

Expand all the fractions:

$$(a+b)^2 = ab(a-b)$$

$$(a+b)ab+(a-b)ab = (a+b)(a-b)$$

Notice the $ab(a-b)$ in both equations, and substitute the first into the second:

$$(a+b)ab+(a+b)^2 = (a+b)(a-b)$$

Clearly $a+b$ is nonzero, so

$$ab+a+b = a-b$$

giving $ab+2b=0$, so if b is nonzero, then $a=-2$ so $a^2=4$.

Problem 14

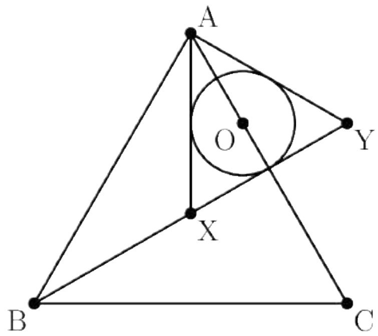
Author: bissue
Problem: Determine the smallest positive integer n such that among the positive integers between n and $(n+10)$, inclusive,

- there are five multiples of 2.
- there are three multiples of 3.
- there are three multiples of 5.
- there is one multiple of 7.

Answer: 85
Solution: Since there are 5 multiples of 2, we know that n is odd. Also, since there are 3 multiples of 3, n must be 1 mod 3. Finally, since there are 3 multiples of 5, n must be divisible by 5. By Chinese Remainder Theorem, n is 25 mod 30. Now, we check which n work. $n = 25$ doesn't work since both 28 and 35 are in the range, $n = 55$ doesn't work since both 56 and 63 are in the range, but $n = 85$ works since only 91 is divisible by 7. Thus, $n = 85$ is the answer.

Problem 15

Author: Depsilon0
Problem: Consider equilateral triangle ABC with side length 6. Let circle ω have radius 1 and center O, and suppose O lies on segment $\overline{AC}$ such that $OA = 2$. Let the tangent from B to ω closer to $\overline{BC}$ intersect the tangents from A to ω at X and Y. Find XY^2.
Answer: 12
Solution: We have the following diagram:



Notice the tangent from B to ω closer to BC intersects the midpoint of AC. Suppose the midpoint of AC is M. We also know this is the altitude.

Suppose ω is tangent to AX at T. Then, $OT = 1$ and $OT \perp AX$. Thus, $\triangle AOT$ is a $30-60-90$ triangle. Since $XM \perp AM$, we know $\triangle AXM$ is a $30-60-90$ triangle. So, $XM = \sqrt{3}$. Similarly, we know $MY = \sqrt{3}$. Thus, $XY = 2\sqrt{3}$. Finally, we obtain $XY^2 = 12$.

Problem 16

Author: Hanson Liu, David Altizio
Problem: Let $f(x)$ be a quadratic with integer coefficients. Suppose there exist positive primes $p < q$ such that $f(p) = f(q) = 87$ and $f(p+q) = 178$. Find $p^2 + q^2$.
Answer: 218
Solution: Define $g(x) = f(x) - 87$, so $g(p) = g(q) = 0$ and thus $g(x) = a(x-p)(x-q)$ for some integer a. Then, $g(p+q) = apq = 178 - 87 = 91$, which forces $a = 1$ and $p, q = 7, 13$ in some order. Thus, the answer is $49 + 169 = 218$.

Problem 17

Author: bissue
Problem: How many permutations of the first 9 positive integers satisfy both of these properties?

- The leftmost even number is 6.
- The rightmost odd number is 9.

Answer: 18144
Solution: By symmetry, $\frac{1}{4}$ of all random permutations have 6 as the leftmost even number and $\frac{1}{5}$ of all random permutations have 9 as the leftmost odd number. Therefore, the answer is $9! \cdot \frac{1}{5} \cdot \frac{1}{4} = 18144$.

Problem 18

Author: Ethan Lee
Problem: Isosceles trapezoid $ABCD$ with $\overline{AB} \parallel \overline{CD}$ has an incircle ω_1 tangent to BC at E. Another circle ω_2 is tangent to ω_1, CD, and also BC at F. If $BE = CF = 1$, and the area of the trapezoid is A, what is A^2?
Answer: 192
Solution: Let the common tangent of the two circles intersect BC at P and CD at Q. By radical axis P is the midpoint of EF and therefore also the midpoint of BC. By symmetry, $PQ = PB = PC$, but also $CP = CQ$ so CPQ is equilateral. This implies that $BQ \perp QC$, so we calculate $BQ = \sqrt{BC^2 - QC^2}$. The area of the trapezoid is the sum of the bases divided by 2 times its height, which can be calculated to be $8\sqrt{3}$ by all the congruent segments, which yields 192 upon squaring.

[1] [2]

[1]The publication, reproduction or communication of the problems or solutions of all JMPSC exams during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

[2]The team on the Junior Mathematician's Problem Solving Competition (JMPSC) reserves the right to re-examine students before deciding whether to grant official status to their scores.