Sprint Round

Junior Mathematicians' Problem Solving Competition

July 10th, 2021

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1. This is a twenty question free-response test. Each question is has exactly one integer answer.
2. You have 50 minutes to complete the test.
3. You will receive 3 points for each correct answer, and 0 points for each problem left unanswered or incorrect.
4. Figures are not necessarily drawn to scale.
5. No aids are permitted other than scratch paper, graph paper, rulers, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. When you finish the exam, please stay in the zoom meeting for further instructions.

1 Problem 1

Compute $\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{9}\right)(3+6+9)$.

2 Problem 2

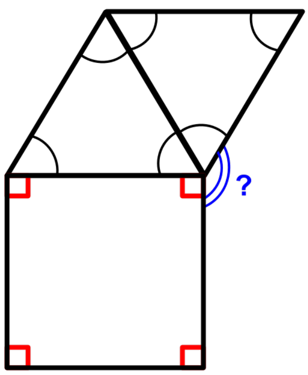
If Samuel has an unlimited supply of quarters ($0.25), dimes ($0.10), nickels ($0.05), and pennies ($0.01), then what is the least number of coins Samuel can use to pay off $1.79?

3 Problem 3

What two-digit even number has digits that sum to 17?

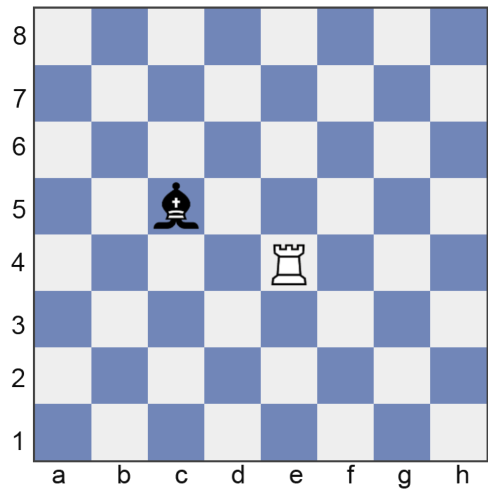
4 Problem 4

If all angles marked with a red square are 90° and all angles marked with one black curve are equal, find the measure of the blue angle indicated by two blue curves and a question mark.



5 Problem 5

A standard chess-board is an 8 by 8 grid of squares. A bishop is placed at square C5 (row 5, column C) and a rook is placed at square E4 (row 4, column E). It is possible for the bishop and rook to visit the same square in one move. How many of these squares exist?



BISHOP: A bishop is a piece that can move to any square lying on the two diagonals passing through it. In this chess board, the bishop can visit A7, B6, G1, D6, and so on.
ROOK: A rook is a piece that can move to any square lying on the same row or column. In this chess board, the rook can visit E5, E8, H4, C4, and so on.

6 Problem 6

How many positive two-digit numbers exist such that the product of its digits is not zero?

7 Problem 7

Brady has 100 coins that are all either nickels or dimes. If the probability of randomly picking a nickel from these 100 coins is $\frac{2}{5}$, how much money does Brady have in dollars?

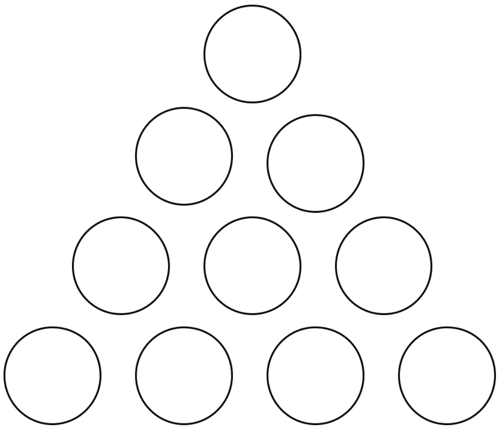
8 Problem 8

How many numbers are in the finite sequence of consecutive perfect squares

$$9, 16, 25, \ldots, 2500?$$

9 Problem 9

How many sets of three circles such that their centers lie on the same line exist in the picture below? Assume that sets of consecutive three-in-a-row circles that appear to have their centers lying on the same line actually do have their centers lying on the same line.



10 Problem 10

Qiao feeds a piece of metal into a machine, which cuts it into pieces at a constant rate. He wants the original piece of metal split into twenty pieces, but after leaving the machine alone for twenty minutes, he finds that it has only been split into six pieces. How many more minutes does Qiao need to wait?

11 Problem 11

If $x\sqrt{x\sqrt{x}} = 2^{15}$, x can be written as $2^{\frac{m}{n}}$, where m and n are relatively prime. What is $m - n$?

12 Problem 12

Yireh gets a grade of 75% for his chapter test. She doesn't remember how many problems there were, but she remembers that there were fewer than 18 problems, each problem solved correctly was worth 1 point, and she did not receive any points for an incorrect or skipped question. Find the sum of all the possible numbers of problems that the test could have had.

13 Problem 13

Abby, Bryant, Cam, and David each tell one truth and one lie.

Abby: Bryant is the tallest among the four of us. Cam is the shortest among the four of us.

Bryant: Abby is the oldest in the room. Abby is the shortest in the room.

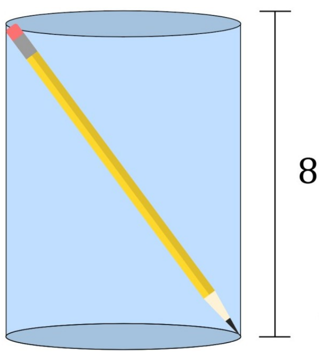
Cam: David is taller than me. David is older than me.

David: I am not the oldest person in the room. My name is David.

If the number of letters in the name of the shortest person is a and the number of letters in the name of the second-tallest person is b, find $a \times b$.

14 Problem 14

Grace places a pencil in a cylindrical cup and is surprised to see that it fits diagonally. The pencil is 17 units long and of negligible thickness. The cup is 8 units tall. The volume of the cup can be written as $k\pi$. Find k.

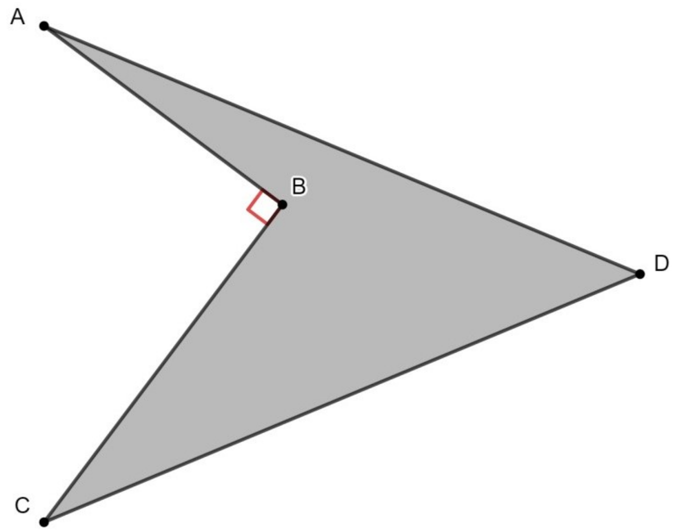


15 Problem 15

Find the last two digits of $10^{10} - 5^{10}$.

16 Problem 16

$ABCD$ is a concave quadrilateral with $AB = 12$, $BC = 16$, $AD = CD = 26$, and $\angle ABC$ is a right angle. Find the area of $ABCD$.



17 Problem 17

What is the smallest positive multiple of 1003 that has no zeros in its decimal representation?

18 Problem 18

As an April Fool's prank, Joseph hacks his teacher's digital clock and switches each digit to a certain letter. Right now, the hacked clock displays **M:AT**. 14 minutes later, it displays **A:TM**. If $A \neq T \neq M$, what digit is the letter **T** supposed to display?

19 Problem 19

On square $ABCD$ with side length 64, M is the midpoint of $\overline{CD}$. Let E be the foot of the altitude from M to $\overline{AC}$. Find AE^2.

20 Problem 20

For all integers x and y, define the operation Δ as

$$x \Delta y = x^3 + y^2 + x + y.$$

Find the positive integer k such that

$$257 \Delta 256 = 258k^2.$$

[1] [2]

[1] The publication, reproduction or communication of the problems or solutions of all JMPSC exams during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

[2] The team on the Junior Mathematician's Problem Solving Contest (JMPSC) reserves the right to re-examine students before deciding whether to grant official status to their scores.