Accuracy Round

Junior Mathematicians' Problem Solving Competition

July 10th, 2021

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1. This is a fifteen question free-response test. Each question has exactly one integer answer.
2. You have 60 minutes to complete the test.
3. You will receive 4 points for each correct answer, and 0 points for each problem left unanswered or incorrect.
4. Figures are not necessarily drawn to scale.
5. No aids are permitted other than scratch paper, graph paper, rulers, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. When you finish the exam, please stay in the Zoom meeting for further instructions.

1 Problem 1

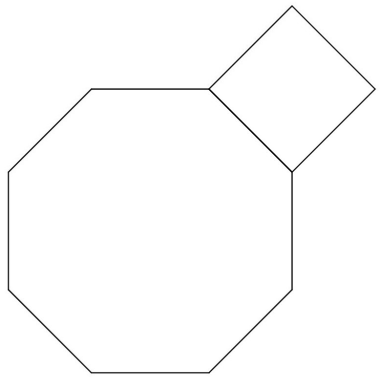
Find the sum of all positive multiples of 3 that are factors of 27.

2 Problem 2

Three distinct even positive integers are chosen between 1 and 100, inclusive. What is the largest possible average of these three integers?

3 Problem 3

In a regular octagon, the sum of any three consecutive sides is 90. A square is constructed using one of the sides of this octagon. What is the area of the square?



4 Problem 4

If $\frac{x+2}{6}$ is its own reciprocal, find the product of all possible values of x.

5 Problem 5

Let $n! = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$ for all positive integers n. Find the value of x that satisfies

$$\frac{5!x}{2022!} = \frac{20}{2021!}.$$

6 Problem 6

In quadrilateral $ABCD$, diagonal $\overline{AC}$ bisects both $\angle BAD$ and $\angle BCD$. If $AB = 15$ and $BC = 13$, find the perimeter of $ABCD$.

7 Problem 7

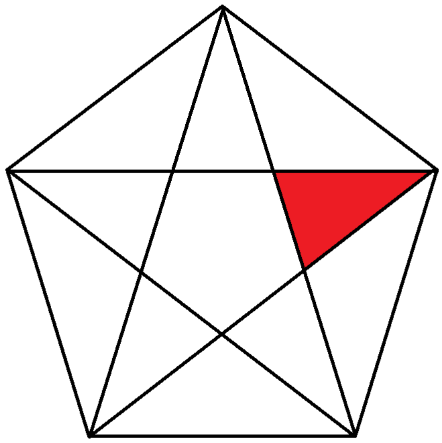
If A, B, and C each represent a single digit and they satisfy the equation

$$\begin{array}{cccc} & A & B & C \\ \times & & & 3 \\ \hline & 7 & 9 & C \end{array},$$

find $3A + 2B + C$.

8 Problem 8

How many triangles are bounded by segments in the figure and contain the red triangle? (Do not include the red triangle in your total.)



9 Problem 9

If $x_1, x_2, \ldots, x_{20}$ is a strictly increasing sequence of positive integers that satisfies

$$\frac{1}{2} < \frac{2}{x_1} < \frac{3}{x_2} < \cdots < \frac{11}{x_{10}},$$

find $x_1 + x_2 + \cdots + x_{10}$.

10 Problem 10

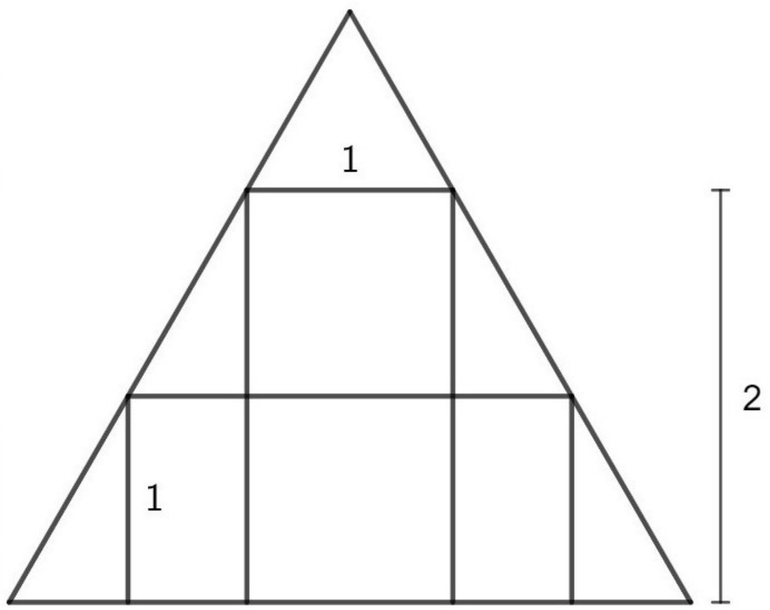
In a certain school, each class has an equal number of students. If the number of classes was to increase by 1, then each class would have 20 students. If the number of classes was to decrease by 1, then each class would have 30 students. How many students are in each class?

11 Problem 11

If $a : b : c : d = 1 : 2 : 3 : 4$ and a, b, c, and d are divisors of 252, what is the maximum value of a?

12 Problem 12

A rectangle with base 1 and height 2 is inscribed in an equilateral triangle. Another rectangle with height 1 is also inscribed in the triangle. The base of the second rectangle can be written as a fully simplified fraction $\frac{a+b\sqrt{3}}{c}$ such that $gcd(a,b,c) = 1$. Find $a+b+c$.



13 Problem 13

Let x and y be nonnegative integers such that $(x+y)^2+(xy)^2 = 25$. Find the sum of all possible values of x.

14 Problem 14

What is the leftmost digit of the product

$$\underbrace{161616\cdots16}_{100\text{ digits}}\times\underbrace{252525\cdots25}_{100\text{ digits}}?$$

15 Problem 15

For all positive integers n, define the function $f(n)$ to output $4\underbrace{777\cdots7}_{n\text{ sevens}}5$. For example, $f(1) = 475$, $f(2) = 4775$, and $f(3) = 47775$. Find the last three digits of

$$\frac{f(1)+f(2)+\cdots+f(100)}{25}.$$

[1] [2]

[1]The publication, reproduction or communication of the problems or solutions of all JMPSC exams during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

[2]The team on the Junior Mathematician's Problem Solving Contest (JMPSC) reserves the right to re-examine students before deciding whether to grant official status to their scores.